AN APPROXIMATE TECHNIQUE FOR ANALYSIS OF SLUG TESTS IN WELLS SCREENED ACROSS THE WATER TABLE

by

Z. Hyder
J.J. Butler, Jr.
C.D. McElwee

Kansas Geological Survey
Open-file Report 93-44

Disclaimer

An Approximate Technique for Analysis of Slug Tests in Wells Screened Across the Water Table

Z. Hyder, J. J. Butler, Jr., and C. D. McElwee
Kansas Geological Survey
1930 Constant Avenue
Lawrence, KS 66047

Prepared for presentation at
The American Geophysical Union
1993 Fall Meeting
San Francisco, Ca.
December 6, 1993

KGS Open File Report #93-44

ABSTRACT

The slug test has become one of the most commonly used techniques for the in-situ estimation of hydraulic conductivity in unconfined flow systems. Although several approaches are available for analysis of data from wells screened below the water table, no methods have been developed for the case of a well screened across the water table, a common configuration at suspected sites of shallow groundwater contamination. An approximate approach for the analysis of data from such wells has been developed through an extension of the Bouwer and Rice model to the case in which the effective screen length changes during the course of the test. This extension, which employs the Dupuit-Forchheimer approximation for free surface flow, is able to reproduce some of the key features seen in field data from slug tests in wells screened across the water table. The most important of these features are that the duration of a slug test (and thus the conductivity estimate) is dependent on the magnitude of the initial displacement and that a log-linear displacement versus time plot is concave upward for an injection slug test and concave downward for a withdrawal slug test. Applications of this approximate model to field data are presented.

INTRODUCTION

The slug test is a popular technique for estimating the hydraulic conductivity of an unconfined aquifer. The method consists of displacing the water level in a well from static in a near instantaneous fashion and then measuring the recovery of water level through time at that well. The test is relatively inexpensive and needs much less time for completion than a pumping test. The slug test is also very convenient for contaminated sites because it can be performed without removing water from the well. For the most part, slug tests are performed in wells that were originally installed for monitoring purposes. In unconfined aquifers, monitoring wells are commonly screened across the water table to detect light non-aqueous phase liquids (LNAPL's) such as gasoline and other hydrocarbons. Thus, slug tests are often performed in wells screened across the water table.

The Bouwer and Rice model (Bouwer and Rice, 1976; Bouwer, 1989) is commonly used for analyzing data from slug tests in unconfined aquifers. This model was specifically developed for the case of the water table located above the top of the screen. Hence, the length of the screen through which water moves in and out of the well is constant with time. However, for slug tests in wells where the water table is in the screen, the length of screen through which significant quantities of water flow (henceforth designated the effective screen length, b-H(t) on Figure 1) will change as the test progresses. This time dependence of the effective screen length makes the Bouwer and Rice method of questionable utility for slug tests in wells screened across the water table.

Field data from slug tests performed in wells screened across the water table often display features that are not accounted for in the Bouwer and Rice model. For example, the shape of a log-linear displacement versus time curve is often concave downwards for a withdrawal slug test when the water table is in the screen. When the water table is above the screen, the same plot will be concave upwards. Similarly, the duration of a slug test will depend on the magnitude of the initial displacement when the water table is in the screen, while test duration is independent of initial displacement when the water table is above the screen. If such effects are not accounted for, they will introduce error into hydraulic conductivity estimates determined from slug-test data. Clearly, new methods are needed for the analysis of slug tests in wells screened across the water table. The development of such a method is the focus of this report.

In this paper, we present a methodology to evaluate withdrawal slug tests performed in wells in which the water table lies within the screen. A solution for injection slug tests can also be derived using a similar mathematical formulation. In earlier work, we have used a semianalytical solution to show the viability of the Bouwer and Rice model for many commonly met field conditions (Butler et al., 1993). Considering the success and simplicity of the Bouwer and Rice model, we decided to base our new model on similar mathematical concepts.

METHODOLOGY

For a withdrawal slug test with the water table in the screen, the steady-state discharge for a particular value of drawdown can be written using the Dupuit-Forchheimer theory (Bear, 1972) as

$$2\pi r K(b-s)\frac{\partial s}{\partial r} = -Q \tag{1}$$

where

Q = steady-state discharge, $[L^3/T]$;

s = drawdown in the aquifer, [L];

r = radial direction, [L];

Ignoring the seepage face, the effective screen length is b-H (see Figure 1) and increases with time as the drawdown decreases.

The initial conditions can be written as

$$s(r_w,0)=H_0 \tag{2}$$

where r_w is the radius of screen. When the well has a gravel pack of high permeability, r_w is the radius of the gravel pack.

The boundary conditions are

$$s(r_w,t)=H(t) \tag{3}$$

$$s(R_e,t)=0 \tag{4}$$

where

R_e = the effective radius, [L].

The effective radius is the radius over which the potential changes during a slug test.

The change in the volume of water in the well is equal to the flow through the screened section of the well:

$$-\pi r_s^2 \frac{dH}{dt} = Q \tag{5}$$

The radius of the well, r_s in (5), equals r_w when there is no gravel pack around the well. When the well has a gravel pack, the equivalent value of r_s is $[(1-n)r_s + n\ r_w]^{1/2}$, where n is the porosity of the gravel pack (Bouwer and Rice, 1976).

ANALYTICAL SOLUTION

Integrating (1) we get the discharge through the screened section of the well as

$$Q=\frac{2\pi KH(b-\frac{H}{2})}{\ln\frac{R_e}{r_w}} \tag{6}$$

Equating equations (5) and (6), we get

$$-\pi r_s^2\frac{dH}{dt}=\frac{2\pi KH(b-\frac{H}{2})}{\ln\frac{R_e}{r_w}} \tag{7}$$

After integrating over H and t, the following expression is obtained

$$\ln\frac{(2b-H_0)/H_0}{(2b-H)/H}=-\frac{2Kbt}{r_s^2\ln\frac{R_e}{r_w}} \tag{8}$$

The drawdown (H) in the well for a withdrawal slug test can be extracted from equation (8) as

$$H=\frac{2b}{1+\frac{2b-H_0}{H_0}\exp\left(\frac{2Kbt}{r_s^2\ln\frac{R_e}{r_w}}\right)} \tag{9}$$

PARAMETER ESTIMATION

An expression for estimating hydraulic conductivity from a withdrawal slug test can be obtained by rewriting (8) as

$$K=-\left(\frac{r_s^2\ln\frac{R_e}{r_w}}{2b}\right)\frac{1}{t}\ln\frac{(2b-H_0)/H_0}{(2b-H)/H} \tag{10}$$

Data analysis using (10) is quite straightforward. Semilog plots of the dimensionless term inside the logarithm versus time are prepared from field data. The slope of a straight line fit to this plot is then used in (10) to estimate hydraulic conductivity. The major issue concerning the outlined approach is what to employ for the $\ln(R_e/r_w)$ term. As a first cut, the empirically determined values of Bouwer and Rice can be used. The justification is that the effective radius will not change greatly from the Bouwer and Rice case when the water table is in the screen.

The approach outlined above is based on a simplified representation of the actual physical system. The viability of this representation has not yet been thoroughly demonstrated. Ongoing numerical simulation work is directed at this objective. Although the issue is not yet resolved, strong support for this approach can be found in the soils and agricultural engineering literature. In the following section, a related technique in agricultural engineering is described.

AUGER-HOLE METHOD

The auger-hole method used by agricultural engineers for measuring saturated hydraulic conductivity is mathematically identical to the case of a slug test in a well screened across the water table. Thus, it is of interest to examine standard approaches for analyzing data from auger-hole tests.

Bouwer and Jackson (1974) and Amoozegar and Warrick (1986) give detailed reviews of auger-hole tests and related methodology used in soil and agricultural engineering. The auger-hole test involves augering a shallow hole (usually 1-2 meters in depth) that extends below the water table. The water level in the hole is suddenly lowered by removing water from the hole. The rate of rise of water level in that hole is then recorded. In general, the auger-hole techniques use the rate of rise of the water level at any time in the auger hole to estimate hydraulic conductivity. Since two consecutive water-level measurements are used to measure the rate of rise, measurement noise may result in large errors. This approach differs from the slug test techniques in groundwater in which all the head measurements are used for parameter estimation.

A commonly used approach for the analysis of auger-hole test data was developed by Ernst (1950) who proposed an equation to estimate hydraulic conductivity for the case of an impermeable boundary at a infinite distance below the bottom of the auger hole

$$K = \frac{4.63 r_w^2}{(b + 20 r_w)(2 - H/b) H} \frac{\Delta H}{\Delta t} \tag{11}$$

This equation can be applied when the distance of the impermeable boundary from the bottom of the hole is greater than 0.5b. van Beers (1958) later modified this equation for the case when the auger-hole terminates at an impermeable surface

$$K=\frac{4.17r_w^2}{(b+10r_w)(2-H/b)H}\frac{\Delta H}{\Delta t} \qquad \textbf{(12)}$$

MODIFIED ERNST FORMULA

In order to compare the Ernst formula (11) to equation (10), equation (11) must be rewritten using the following approximation

$$\frac{\Delta H}{\Delta t}=-\frac{dH}{dt} \qquad \textbf{(13)}$$

The minus sign in (13) arises from the different definition of H used by the two approaches (deviation from static in (10) versus length of column of water in auger hole in (11)). Equation (11) can now be rewritten as

$$-\pi r_w^2\frac{dH}{dt}=\frac{2\pi KH(b-\frac{H}{2})}{[4.63b/(b+20r_w)]} \qquad \textbf{(14)}$$

A comparison of (7) and (14) indicates the only difference is the denominators of the right hand sides of (7) and (14), both of which are empirical constants.

$$C = \ln\left(\frac{R_e}{r_w}\right) = \frac{4.63b/r_w}{(b/r_w + 20)} \tag{15}$$

Note that (14) is based on the assumption of negligible water table movement in the aquifer.

EVALUATION OF EFFECTIVE RADIUS

In order to compare the empirical constants employed by the two approaches, an effective radius must be found for use in equation (10). As a preliminary step, we used the results of Bouwer and Rice (1976) for the case of the water table above the top of the screen. Although the effective radius is not expected to be the same, the hydraulic conductivity estimates will not be greatly affected because the effective radius appears within a logarithmic term. The equations to determine $[\ln(R_e/r_w)]$ are given in Bouwer and Rice (1976) and Bouwer (1989). The equations are rewritten by putting the depth from the water table to the screen bottom as the screen length. In our notation, the equation for a partially penetrating well is

$$\ln\frac{R_e}{r_w}=\left[\frac{1.1}{\ln(b/r_w)}+\frac{A+B\ln[(D-b)/r_w]}{b/r_w}\right]^{-1} \qquad \textbf{(16)}$$

where D is the saturated depth of the aquifer.

The corresponding equation for a well in which the screen abuts against an underlying layer (designated as a fully penetrating well by Bouwer and Rice) is

$$\ln\frac{R_e}{r_w}=\left[\frac{1.1}{\ln(b/r_w)}+\frac{C}{b/r_w}\right]^{-1} \qquad \textbf{(17)}$$

where A, B, and C are dimensionless constants found from plots given in Bouwer and Rice (1976).

COMPARISON OF CONSTANTS

The hydraulic conductivity estimates obtained from (10) will depend on the constants used in the formula. Figure 2 displays a comparison of the conductivity estimates obtained using the Ernst (eqns. (11) and (12)) and Bouwer and Rice constants (eqns. (16) and (17)) as a function of the aspect ratio (b/r_w). The distance between the bottom of the screen to the impermeable boundary is fixed at 0.5b for (16). The curves indicate that the hydraulic conductivities estimated using the different constants should be within 25 percent of one another.

THE SHAPE OF THE SLUG-TEST RESPONSE

Simulated log(H) versus time plots computed from equation (9) will be concave downwards for a withdrawal slug test. An analogous solution for the injection slug test case shows that the curve will be concave upwards in that case. This is consistent with field results such as those shown in Figure 3 (from Dahl and Jones, 1993). It should be emphasized that log-linear data plots for slug tests in wells with the water table above the screen will be concave upwards.

DEPENDENCE ON INITIAL HEADS

Parameters estimated from slug tests performed in wells in which the water table is above the screen are independent of the magnitude of the initial displacement (H_0). Hence, normalized log(H/H_0) versus time plots for different initial displacements will be indistinguishable from one another. However, when the water table is in the screen, simulations performed using equation (9) demonstrate that the duration of the slug test will depend on H_0. In this case, hydraulic conductivity estimates obtained using the Bouwer and Rice model will be a function of H_0. Field data that display such a dependence are shown in Figure 4 (Dahl and Jones, 1993). Table 1 summarizes the differences in Bouwer and Rice hydraulic conductivity estimates that were reported for the tests shown in Figures 3 and 4.

Table 1

Test type	Initial Drawdown	Bouwer and Rice K estimates
Withdrawal	3.51 ft	1.82 ft/d
"	5.23 ft	1.53 ft/d
Injection	3.51 ft	8.21 ft/d
"	5.23 ft	6.16 ft/d

One notable characteristic of slug-test responses in these situations can be seen from Figure 4. If we ignore measurements when the normalized drawdown (H/H_0) is less than 0.1, the response curves will be linear and parallel in the mid-time range (between 0.4 and 0.1 in Figure 4). Therefore, K estimates will be identical when the Bouwer and Rice approach is used for estimation in these intervals. An explanation of this behavior is that the water level in the well has sufficiently recovered so that the saturated section (b-s) approaches the initially fully saturated section of the screen (b). Therefore, when using the Bouwer and Rice model, the midtest straight line section of the curve should always be analyzed. It should be emphasized that the above behavior is not in any way related to a high permeability well skin.

AN EXAMPLE

Data from a withdrawal slug test performed in a well with the water table in the screen is analyzed to illustrate several of the above points. The saturated screen length (b) and the saturated depth of the aquifer (D) are 6.5 ft. The screen abuts against a lower impermeable boundary and r_s and r_w are 0.180 ft and 0.302 ft, respectively.

Figure 5 shows a plot of the test data and the best fit for the model presented here. The y-axis on Figure 5 is the dimensionless term $(((2b-H_0)/H_0)/((2b-H)/H))$ in (10). Hence, a straight line can be fit to the plot. The slope of this straight line is then used in (10) to estimate K. The hydraulic conductivity estimate obtained from the Bouwer and Rice model is 0.37 ft/d, while an estimate of 0.58 ft/d is obtained using the model presented in this paper (using (10) with Bouwer and Rice empirical constants). The lower hydraulic conductivity estimate from the Bouwer and Rice method can be attributed to the use of an overly long screen length in that model when compared to the average effective screen length of the test.

CONCLUSIONS

An approximate model was developed to estimate hydraulic conductivity from slug tests performed in wells in which the water table lies within the screen. Observed behavior such as concave downwards semilog displacement versus time plots and dependence of test duration on initial displacement (H_0) are explained by this model. Field data and related methods in agricultural engineering provide support for the proposed approach. Ongoing numerical work is directed at assessing the general viability of this approximate model.

ACKNOWLEDGEMENTS

The activities on which this paper is based were financed in part by the Department of the Interior, U.S. Geological Survey, through the Kansas Water Resources Research Institute. The contents of this

publication do not necessarily reflect the views and policies of the Department of the Interior, nor does mention of trade names or commercial products constitute their endorsement by the United States government.

REFERENCES

Amoozegar, A., and A. W. Warrick, 1986, Hydraulic conductivity of saturated soils: Field methods, in: Methods of Soil Analysis, Part 1, Agronomy, 9, ed. A. Klute, 735-770.

Bear, J. 1972. Dynamics of fluids in porous media, American Elsevier, New York, 567 pp.

Bouwer, H., 1989, The Bouwer and Rice slug test - an update, Ground Water, v. 27, no. 3, 304-309.

Bouwer, H., and R. D. Jackson, 1974, Determining soil properties, In: Drainage for agriculture, J. van Schilfgaarde (ed.), Agronomy 17: 611-672.

Bouwer, H. and R. C. Rice, 1976, A slug test for determining hydraulic conductivity of unconfined aquifers with completely or partially penetrating wells, Water Resour. Res.,v. 12, no. 3, 423-428.

Butler, J. J., Jr., C. D. McElwee, and Z. Hyder, 1993, Slug tests in unconfined aquifers, Kansas Water Resources Research Inst., Contribution No. 303, 67 pp.

Dahl, S., and J. Jones, 1993, Evaluation of slug test data under unconfined conditions with exposed screens, and low permeability filter pack, Proceedings of the Seventh National Outdoor Action Conference, Ground Water Management, v. 15, 609-623.

Ernst, L. F., 1950, A new formula for the calculation of the permeability factor with the auger hole method, T.N.O., Groningen, The Netherlands, Translated from the Dutch by H. Bouwer, Agr. Eng. Dept., Cornell Univ., Ithaca, NY.

van Beers, W. F. J., 1958, The auger hole method, Int. Inst. Land Reclam. and Improvement, Bull no 1, Wageningen, The Netherlands.

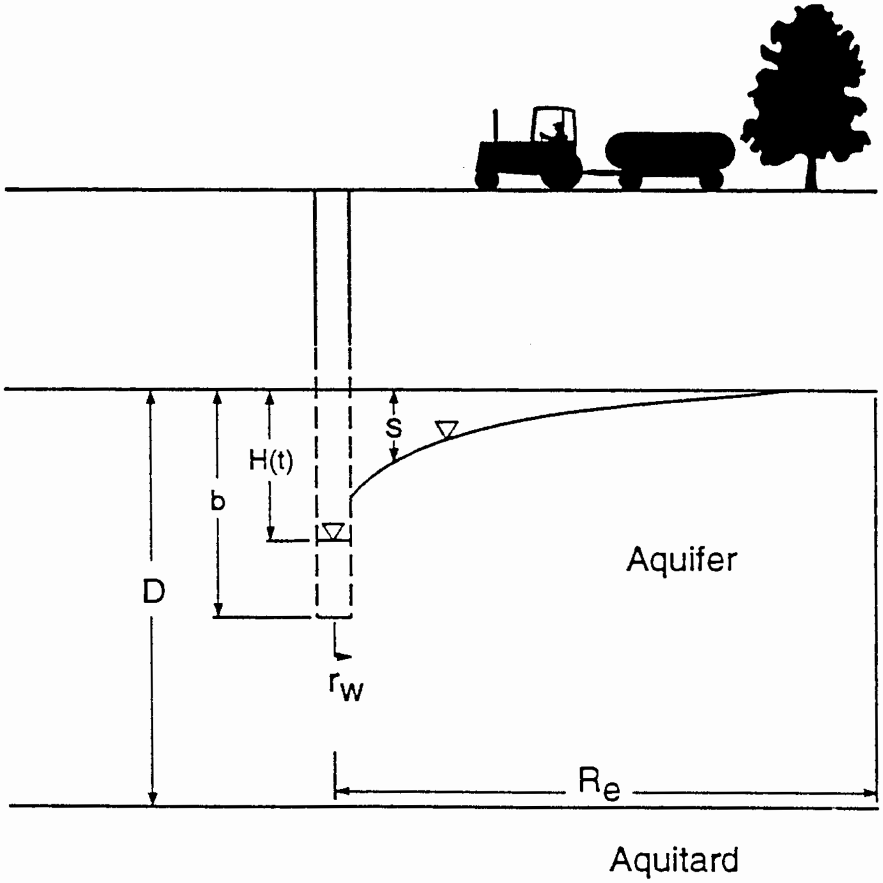

Figure 1. Cross-sectional view of a hypothetical unconfined aquifer.

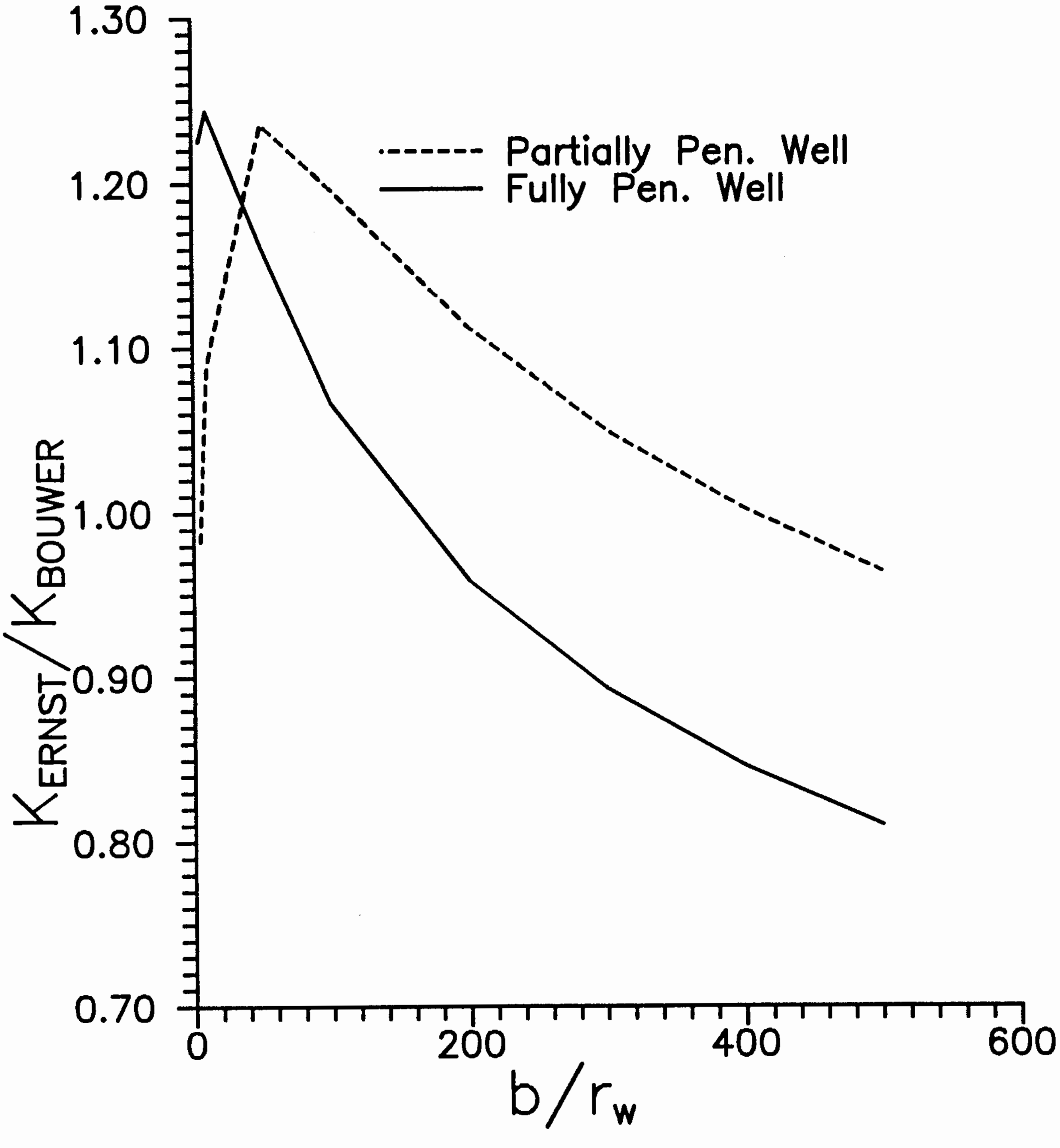

Figure 2. The ratio of hydraulic conductivity estimates obtained using Ernst and Bouwer and Rice constants.

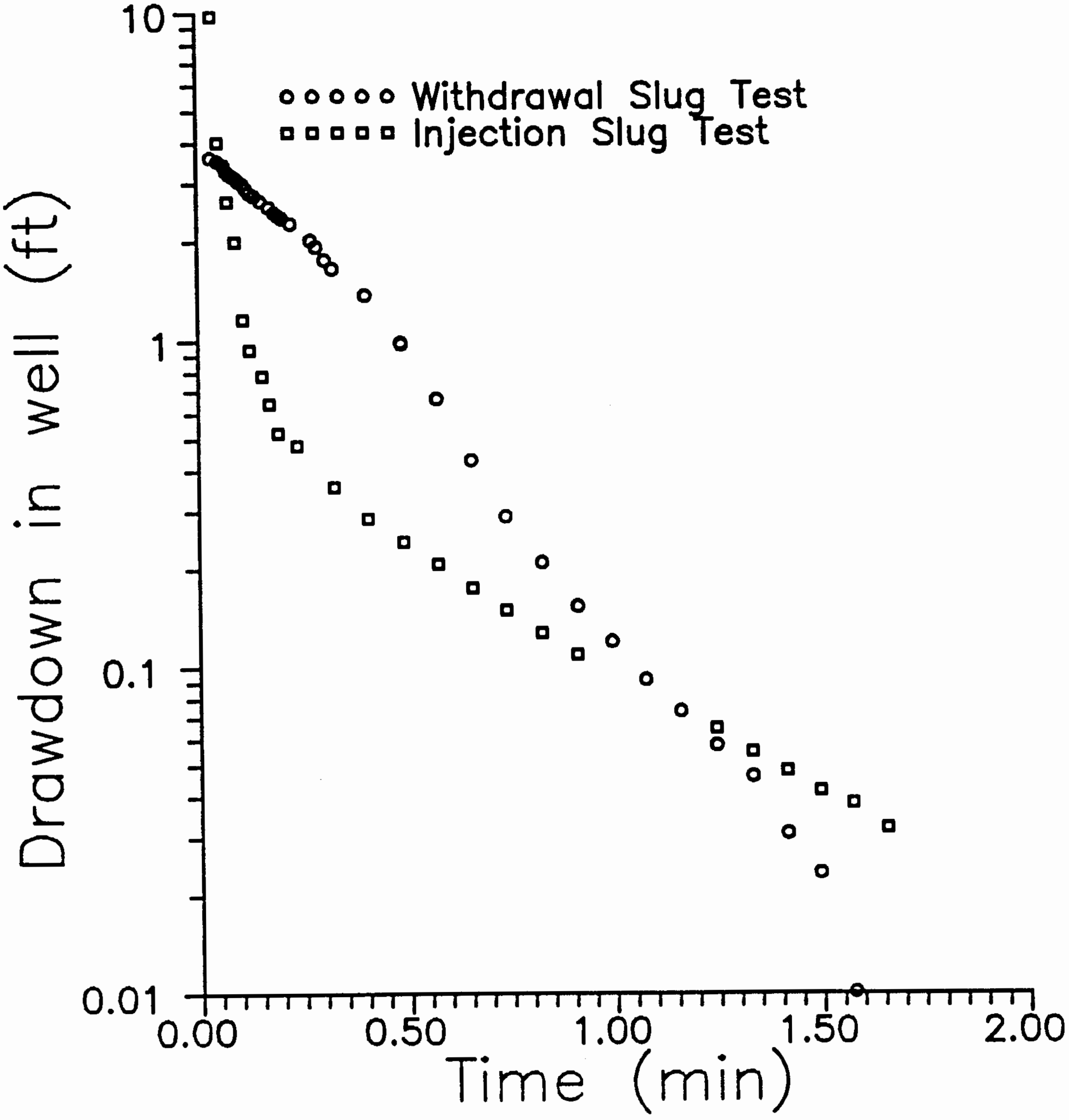

Figure 3. The shape of the responses for withdrawal and injection slug tests.

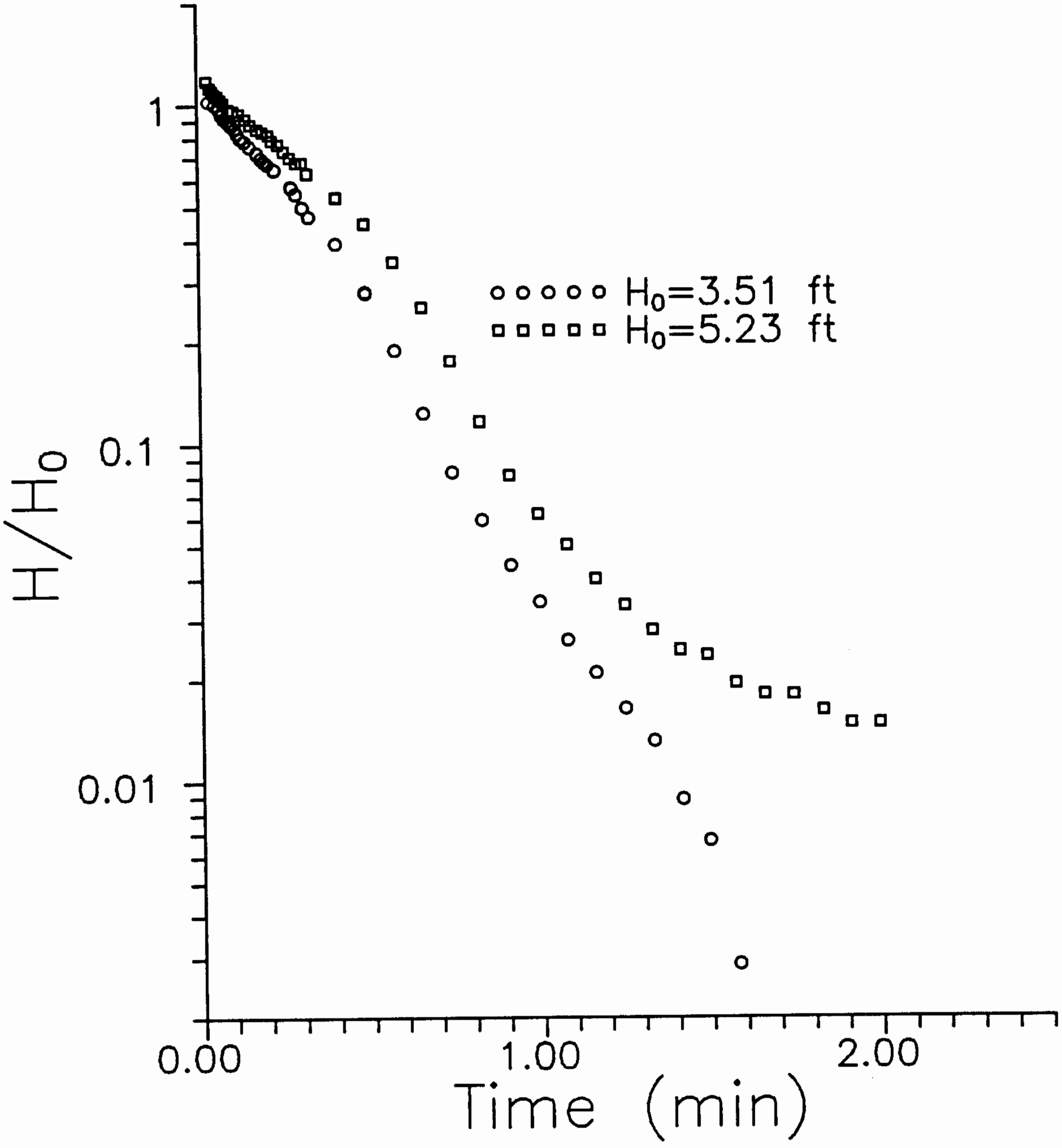

Figure 4. Withdrawal slug-test responses with two different initial heads.

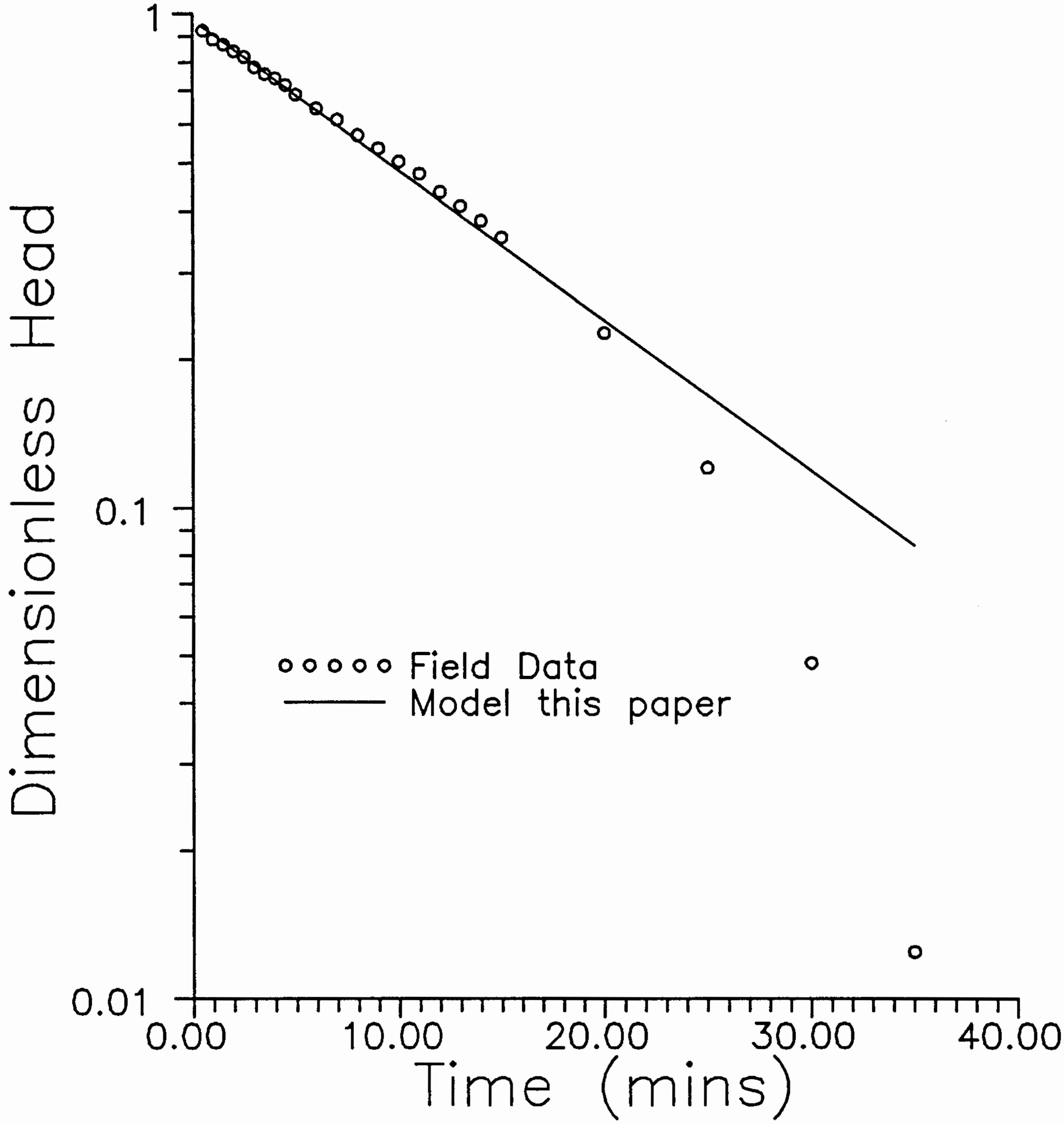

Figure 5. Plot of data from a withdrawal slug test and the best-fit model.