Experimental Verification of a General Model for Slug Tests

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Abstract

Slug tests are widely used to characterize the hydraulic conductivity of a formation. Theoretical models that are linear in form are usually applied to analyze data from slug tests. However, we have observed slug test data at some of our research sites in coarse sand and gravel aquifers that exhibit dependence on initial head and show a systematic lack of fit when analyzed with traditional linear models. In order to analyze these data, we have developed a model (McElwee and Zenner, 1993) that is nonlinear (which explains the dependence on initial head) and allows analysis of data in the underdamped, critically damped, and overdamped regions with no significant lack of fit. The purpose of this paper is to report the application of this model to experimental data that cover a wide range of hydraulic conductivities and a wide range of experimental conditions. We find that the proposed model performs quite suitably over these wide ranges, while reducing to traditional linear models (linear oscillatory, Hvorslev, etc.) when appropriate. This model is of particular use in conditions where traditional linear methods simply can not be meaningfully applied. The proposed model has three parameters: ß which is related to radius changes in the water column, A which is related to the nonlinear head losses, and K the hydraulic conductivity which is inversely related to the linear head losses. We find that the model is quite robust in its estimates of K over varying conditions. Although, ß and A are somewhat empirical and can not be characterized by basic physics as completely as would be desired, we have enough experimental data to understand the basic controls on these two parameters. In summary, this model will allow a wide range of slug test data to be analyzed with a greater accuracy than traditional linear methods.

Introduction

Slug tests are frequently used to characterize the transmissivity of an aquifer. In highly permeable aquifers, however, problems arise when conventional analytical techniques are applied. At one of our field sites in an aquifer consisting of coarse sand and gravel (alluvium) overlain by silt and clay (GEMS - Geohydrologic Experimental and Monitoring Site), we have consistently seen deviations from the expected response of linear theoretical models. Typically, we see a systematic lack of fit to traditional models and a dramatic dependence of the slug test on the magnitude of the initial displacement (Figures 1 and 2). The transient spike seen at very early time is caused by water hammer and is ignored.

Figure 1 shows some typical slug test data from a GEMS well that does not oscillate, but for which the conventional theories do not offer an adequate explanation. The main problems shown in the data of Figure 1 are: 1) the response is dependent on the initial head and 2) the Hvorslev (1951) and Cooper, Bredehoeft, and Papadopulos (1967) models show a systematic lack of fit. In all linear theories the normalized responses for various initial slug heights should collapse onto one curve. Clearly, this is not the case in Figure 1.

In some wells we have also observed oscillatory behavior (Figure 2). Although several authors have developed techniques for analysis of oscillatory data (e.g., Krauss, 1974; van der Kamp, 1976; Kipp, 1985), most of this work has been based on a linear theory. Kabala et al. (1985) are among the first to consider the use of a nonlinear equation to describe the oscillatory slug test behavior. However, after considerable numerical study, they conclude that "the linear model is sufficiently accurate in all practical cases." Stone and Clark (1993) have applied a nonlinear equation to hydraulic work with glaciers.

The General Model for Slug Tests

The motion of the water in the borehole can be described by the Navier-Stokes equations (Eskinazi, 1967). Figure 3 shows a schematic of a typical borehole. If we consider the borehole as a stream tube with average flow in the z direction and integrate over the length of the borehole and use Hvorslev assumptions to describe the aquifer we obtain (McElwee and Zenner, 1993)

$$(h+z_o+b+\beta)\frac{d^2h}{dt^2}+\pi g r_c^2 A\left(\frac{dh}{dt}\right)^2 + g t_o\left[M(h+z_o+b)+1\right]\left(\frac{dh}{dt}\right)+gh=0 \qquad (1)$$

where t_o is the Hvorslev time lag given by

$$t_o=\frac{\pi r_c^2}{FK} \qquad (2)$$

The quantities h, Z_o, and b are defined in Figure 3. F is the usual Hvorslev form factor and M is given by

$$M=\frac{8\mu}{g t_o r_c^2 \rho} \qquad (3)$$

where μ is viscosity, ρ is density, r_c is the casing radius, and g is the acceleration of gravity. The remaining quantities in equation (1) are β, A, and K, which are taken to be fittable parameters. K is the aquifer hydraulic conductivity and is of the most interest in this paper.

A (sec^2/ft^3) is an empirical parameter that is related to nonlinear flow in the borehole and the aquifer. Some investigators have tried to calculate it from first principles and usually assume it is small. However, our experimental data dictates its presence with a magnitude that is larger than might be expected. As used in equation (1) it is assumed that the source of A is at a localized position in the borehole. This will be the case in most cases where this zone is the packer or the screen. However, in some situations the source of A is perhaps uniformly distributed along the borehole. This will usually be the case when no packer or radical radius changes are present. In these cases, A is replaced with A(h + Z_0 +b) and A will have different units (sec^2/ft^4). β is a correction to the effective length of the water column related to radius changes in the borehole. Referring to a simple case shown in Figure 4, it can be shown that β is given by

$$\beta = Z_B\left[1 - \frac{r_1^2}{r_2^2}\right] \tag{4}$$

For more complicated radius changes expression (4) is modified by simply adding a term for each radius change. We have found that it is difficult to always predict β deterministically, so we use it as a fitted parameter. In summary equation (1) represents our general model with 3 fitted parameters: β, A, and K.

Data Analysis

The model represented by equations (1)-(4) has three parameters (β, A, K) which may be adjusted to fit the field data. We have had good results fitting this model to the GEMS data. Figures 5 and 6 show the fitted theoretical values as stars on the field data plots. All slug tests in two inch wells are conducted with a two inch packer with a one inch central pipe unless otherwise stated. The theory describes the head dependence and general shape of the field data very well. Both the non-oscillatory (Figure 5) and oscillatory (Figure 6) data are predicted very well with the fitted values. Field data for a variety of initial slug heights are reproduced well for a single set of parameters (β, A, K).

Figure 7 presents data from another well at GEMS that is described very well by the present model. All the wells shown in Figures 5-7 are located at one nest and completed at different depths. Well 0-2 is at a depth of 46 feet and exhibits a K of .0022 ft/sec. Well 0-7 is completed at about 55 feet and indicates a K of .0056 ft/sec. Finally, well 0-5 ends at about 65 feet and appears to have a K of .0017. These data indicate that the most conductive zone is not at the base of the alluvium but is around 55 feet below the surface. This is in qualitative agreement with a tracer test that was run at the GEMS site within about 100 feet of nest 0. Another interesting fact is that the packer configuration was the same for all the tests in Figures 5-7 and the tests were all run within two days. This indicates that the aquifer is contributing to the values of β and A, whereas the basic hydraulic theory of the borehole would suggest that β and A should be constant for the same borehole geometry.

Figure 8 presents examples of data taken three years apart at well 0-7. The static water level was about 8 feet higher in 1993 due to an unusually wet summer. The two data sets have similar values of initial head but show quite different decay behaviors because of the different water column lengths. The present model was able to predict this difference in behavior very well and yield almost identical values for K. The value for A varies considerably between the two tests because the packer configuration was changed considerable in the three year period, with the 1996 configuration being much cleaner.

An additional well, 0-9, at nest 0 was tested in 1996. This well is a four inch well whereas the wells in Figures 5-8 are two inch wells. Slug tests at this well were done with a four inch packer with a two inch center pipe. However, well 0-9 is completed at about 57 feet, nearly the same depth as well 0-7. The results of slug testing and analysis are displayed in Figure 9 and show several interesting things. First of all, the value for β is a large negative number. It turns out that this is just the value needed to make the first term in equation (1) go nearly to zero. In other words, inertial effects are not too important here. Second the value for A is relatively small, this is indicated by the data for various initial heads being only slightly separated. In other words, nonlinear effects are not too important here. The Hvorslev model would work relatively well here with only a small spread in K values for the various tests. Lastly, the K value of .0054 ft/sec is very consistent with the value of .0056 ft/sec obtained from 0-7 at about the same depth.

Further testing was carried out at well 0-9 by using a two inch riser pipe in conjunction with a four inch packer. The results of conducting the slug tests in the two inch riser is shown in Figure 10. Notice that the tests are now oscillatory, due to the change in effective casing radius, and that the model predicts this behavior. The value of β is negative as predicted by equation (4) for the situation shown in Figure 4. However, β is not the right magnitude to cancel the effect of the inertial term, it simply lessens its effort from what it would be if the casing was all two inch. The value of A here is per unit length of water column distributed over the length of the two inch section (the packer with two inch central pipe looks like part of the two inch borehole). The K value of .0058 ft/sec is very compatible with the estimates given in Figures 6, 8, and 9 for this depth, with very different conditions. Notice that Figure 10 predicts significant nonlinear effects in two inch wells when a high K is present even without a packer in place.

Figure 11 shows the field data and the resulting model fit for slug tests done at another well nest at GEMS. Well 00-1 is completed at a depth of about 56 feet. This well exhibits the highest hydraulic conductivity (.010 ft/sec) measured at GEMS to date. It is at about the same depth as the maximum hydraulic conductivity measured at nest 0. The theory seems to explain the data of Figure 11 very well. The slug test data of Figure 11 was taken using a clean configuration of the two inch packer, which is shown as (a) in Figure 12. We were curious how the configuration of the packer would affect the results at well 00-1. Figure 12 also shows some other attachments that can be used with the packer: (b) is a 3/4 inch adaptor that can be used to attach additional packers or equipment; (c) is a three foot length of 3/4 inch pipe that can be attached to the adaptor; and (d) is a three foot length of 1/2 inch pipe that can also be attached to the adaptor.

Figure 13 shows data taken in four different configurations of the packer. The clean packer configuration is oscillatory in nature while the configuration with the 1/2 inch pipe is overdamped. The current model fits all the data very well. The results of that analysis are shown in Table 1. Clearly, the parameters β and A are increased by the more complex configurations of the packer. This of course is expected from equation (4) for β. It is also seen from Table 1 that the measured hydraulic conductivity decreases with the more complex packer configurations. Clearly, the packer has some intrinsic conductivity that affects the measured result. Looking closely at equation (1) will show that multiple sources for β and A (packer, borehole, and aquifer) will add algebraically to give the total effective value. On the other hand, multiple sources of K (packer, borehole, and aquifer) will cause the effective K to be given by

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \qquad (5)$$

where K_e is the effective conductivity and K_1, K_2, and K_3 are sources of inverse conductivity or resistance. The results of Table 1 indicate that the hydraulic conductivity of the aquifer at well 00-1 must be at least .010 ft/sec. However, the question comes as to whether the clean packer configuration is still causing some error in estimating K for the aquifer and if so how much?

In order to estimate the intrinsic conductivity of the packer we used a double column apparatus, schematically shown in Figure 14, attached to the side of the KGS building. Using this apparatus we were able run tests with and without the packer present, since the test can be initiated with the valve in one arm. One section of the left arm may contain a filter medium for which we are trying to determine a K. If no medium is present we are simply determining the intrinsic conductivity of the pipes which is probably uniformly distributed over the wetted length of the apparatus. Equation (1) needs to be modified slightly to describe the double column experiment. The driving force is the difference in water level in the two columns and the final static level after a test will depend on the initial head difference in the two columns. After each test the starting static level is brought back to the same level, the valve is closed and additional water is added to the left column to start a new test. Figure 15 shows the data from one run with no packer and no filter medium and the theoretically predicted values obtained by model fitting. The theory seems to describe the experiment very well and the fitted K value is .056 ft/sec. This K value is 5-6 times greater than anything we have observed in the field and indicates that the theory seems to be working fine for K's at least this large.

Table 2 lists the fitted results for four sets of tests: the double column apparatus with and without the packer when no filter medium is present; and the double column apparatus having large rounded river pebbles as the filter medium with and without the packer. Figure 15 shows the results for the first test; the other three tests are described equally well by the present model. Realizing that contributions to β and A are additive and that contributions to K add according to equation (5), it is possible to calculate intrinsic values for the packer system. β appears to be in the 6-7 feet range and A is in the 21-24 sec^2/ft^3 range. The intrinsic K of the packer seems to be in the range of .06-.08 ft/sec. This means that for aquifer conductivities of .006-.008 ft/sec or less the error introduced by the packer will be less than 10%. Of the field data presented in this paper, only well 00-1 is showing a higher conductivity than the .006-.008 ft/sec range. Therefore, the error in aquifer conductivity for well 00-1 may be a little more than 10% while the other values should be 10% or less.

Conclusions

We have shown that slug tests in highly permeably aquifers can exhibit nonlinear behavior. The effects of nonlinearities and inertia, can be quite important. The nonlinear terms make slug test results dependent on the initial head, inertial effects are important when oscillatory behavior is observed, and radius variations in the borehole cause the effective water column length to be greater than expected. We have developed a general model incorporating all these features. This general model can reduce to a Hvorslev type model when nonlinearities and inertial effects are insignificant. We find that the model is quite robust in its estimates of K over varying conditions, including varying static water levels, varying radii, and varying packer configurations . Although, ß and A are somewhat empirical and can not be characterized by basic physics as cleanly as would be desired, we have enough experimental data to understand the basic controls on these two parameters. In summary, this model will allow a wide range of slug test data to be analyzed with a greater accuracy than traditional linear methods. Our results also suggest that nonlinear effects may be important in two inch wells even if a packer is not used when a high K is present.

Acknowledgment

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Table 1
Results of tests on well 00-1 with various packer configurations.

Packer Configuration	β (ft.)	A (sec^2/ft^3)	K (ft/sec)
1" Clean Packer	.504	29.8	.0101
Packer Plus 3/4" adaptor	3.93	65.0	.00669
Packer, adaptor, 3'of 3/4" pipe	13.8	131.	.00560
Packer, adaptor, 3' of 1/2" pipe	92.3	818.	.00198

Table 2
Estimates of the intrinsic conductivity of the clean 1" packer system from the double column experiments.

	U-tube, No Packer	U-tube, With Packer	Packer Values
No Filter Medium			
β (ft)	1.74	8.43	6.69
A (sec^2/ft^3)	1.17	24.6	23.4
K (ft/sec)	.0556	.0333	.0830
Large River Pebbles as Filter Medium			
β (ft)	4.50	11.5	7.00
A (sec^2/ft^3)	81.9	103.	21.1
K (ft/sec)	.0357	.0222	.0587

Figure 1.

Slug Test Response at GEMS Well 0-2

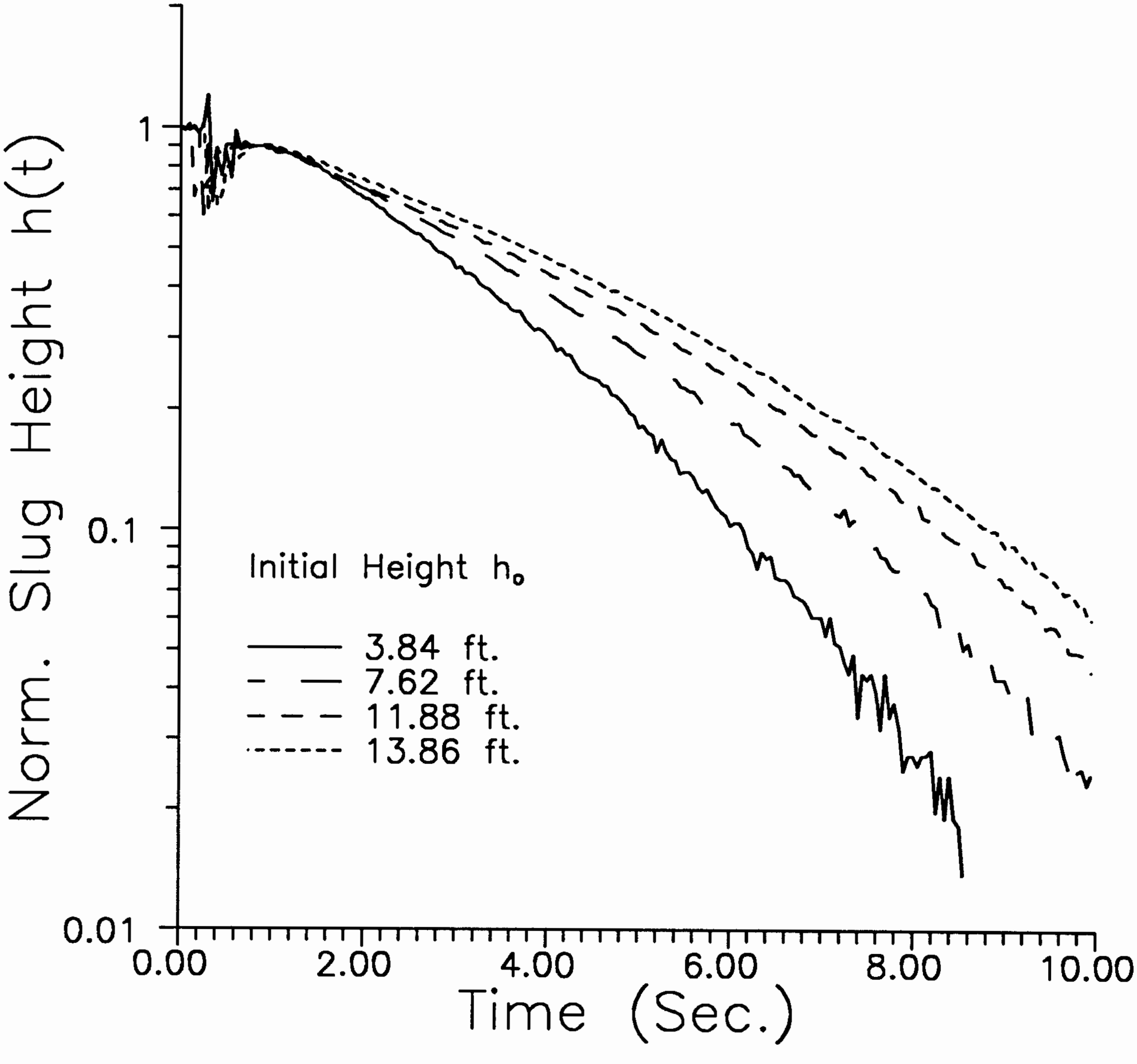


Figure 2.

Slug Test Response at GEMS Well 0-7

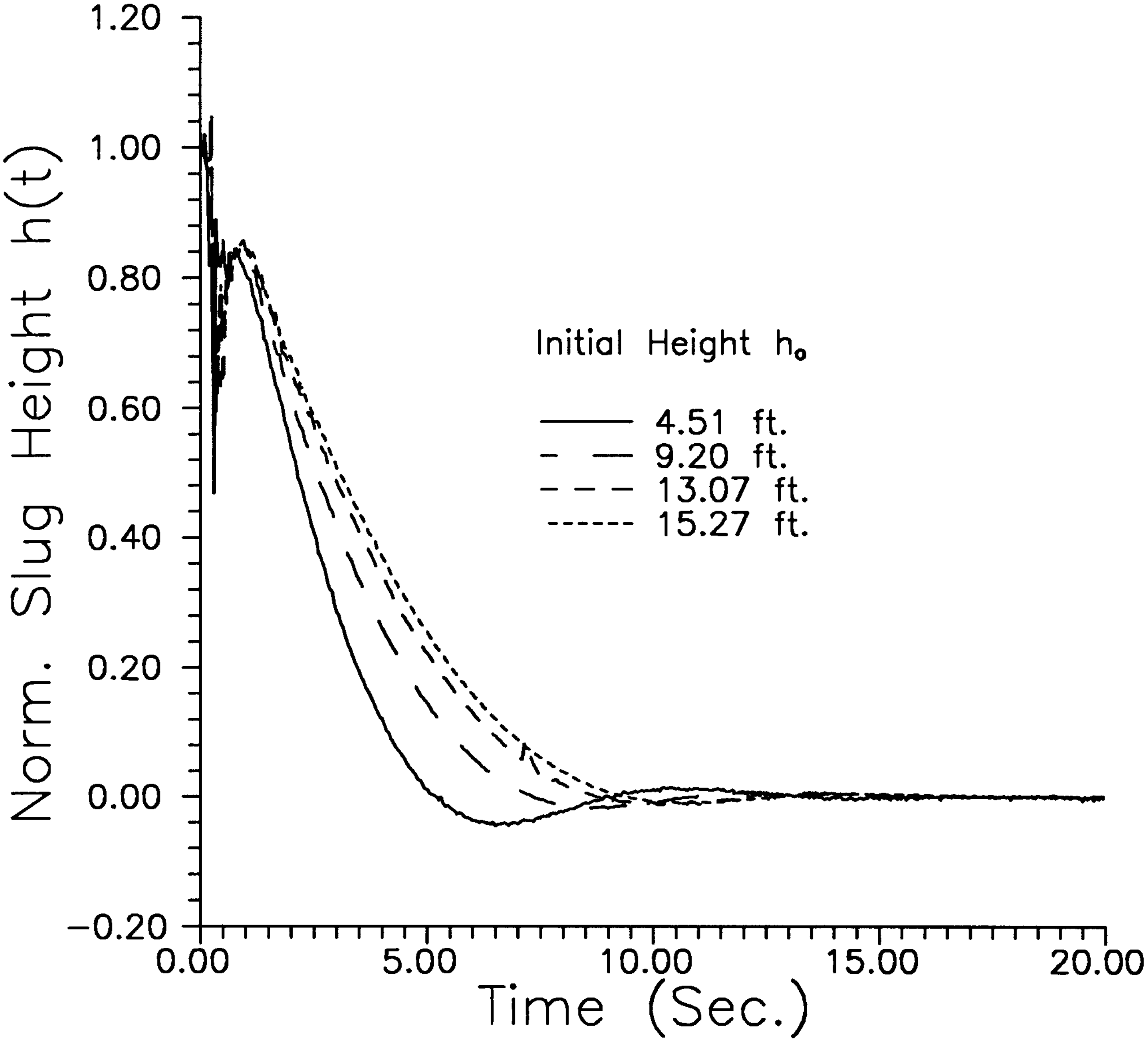


Figure 3. Schematic of the Slug Test Wellbore

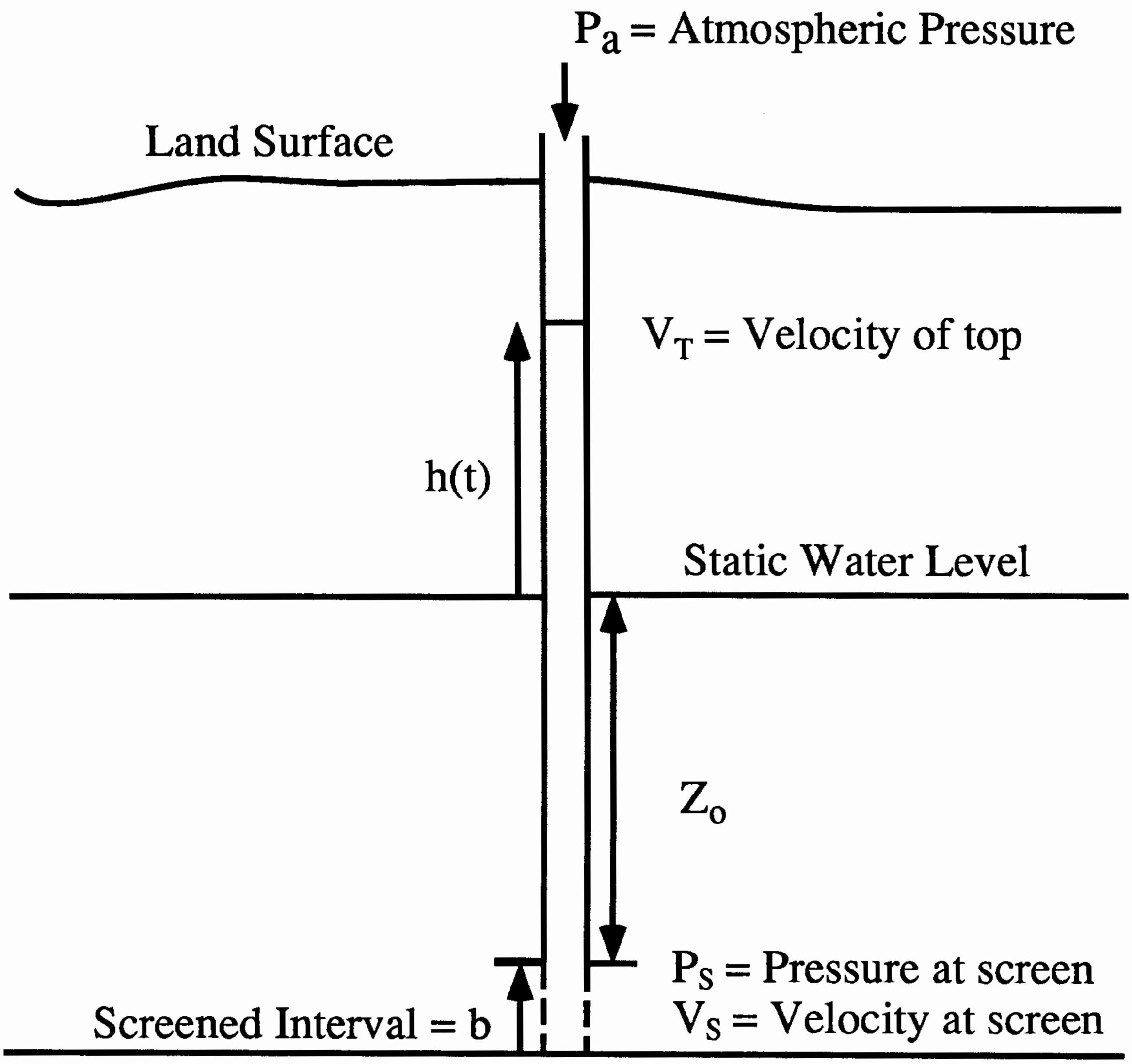


Figure 4

Slug tests with changing casing radius.

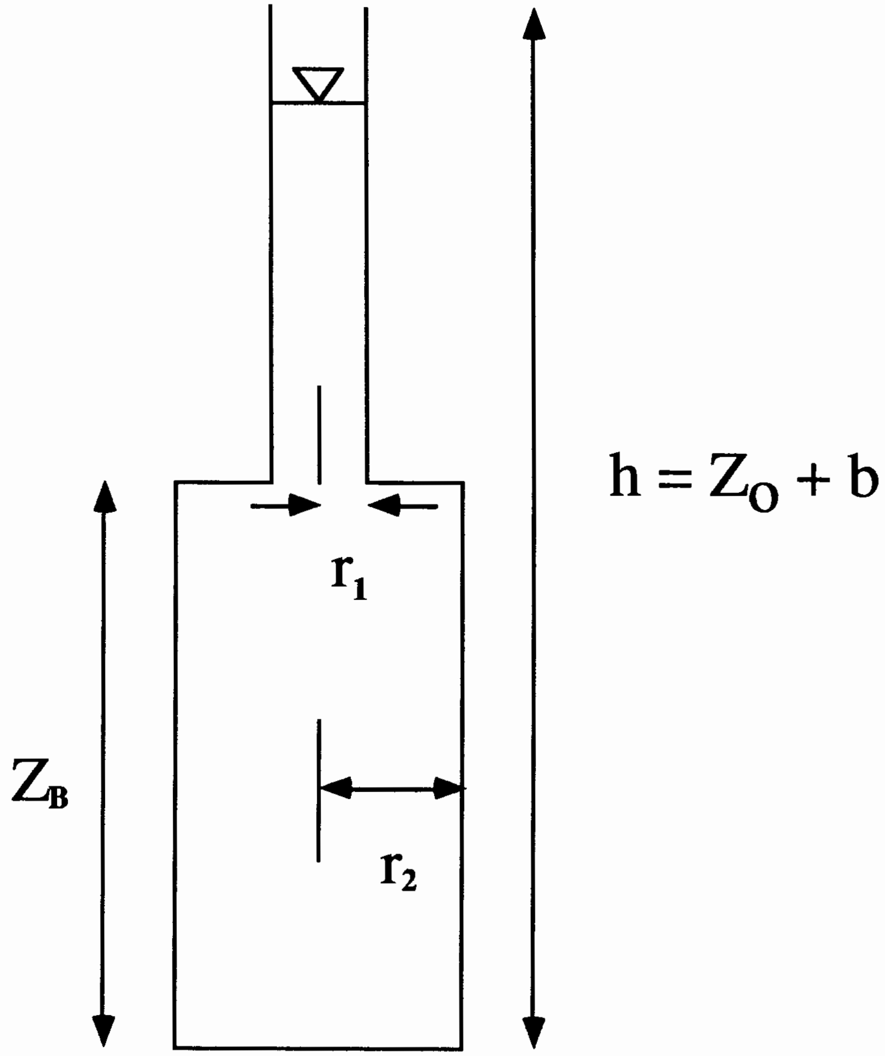


Figure 5.

Slug Test Response at GEMS Well 0–2

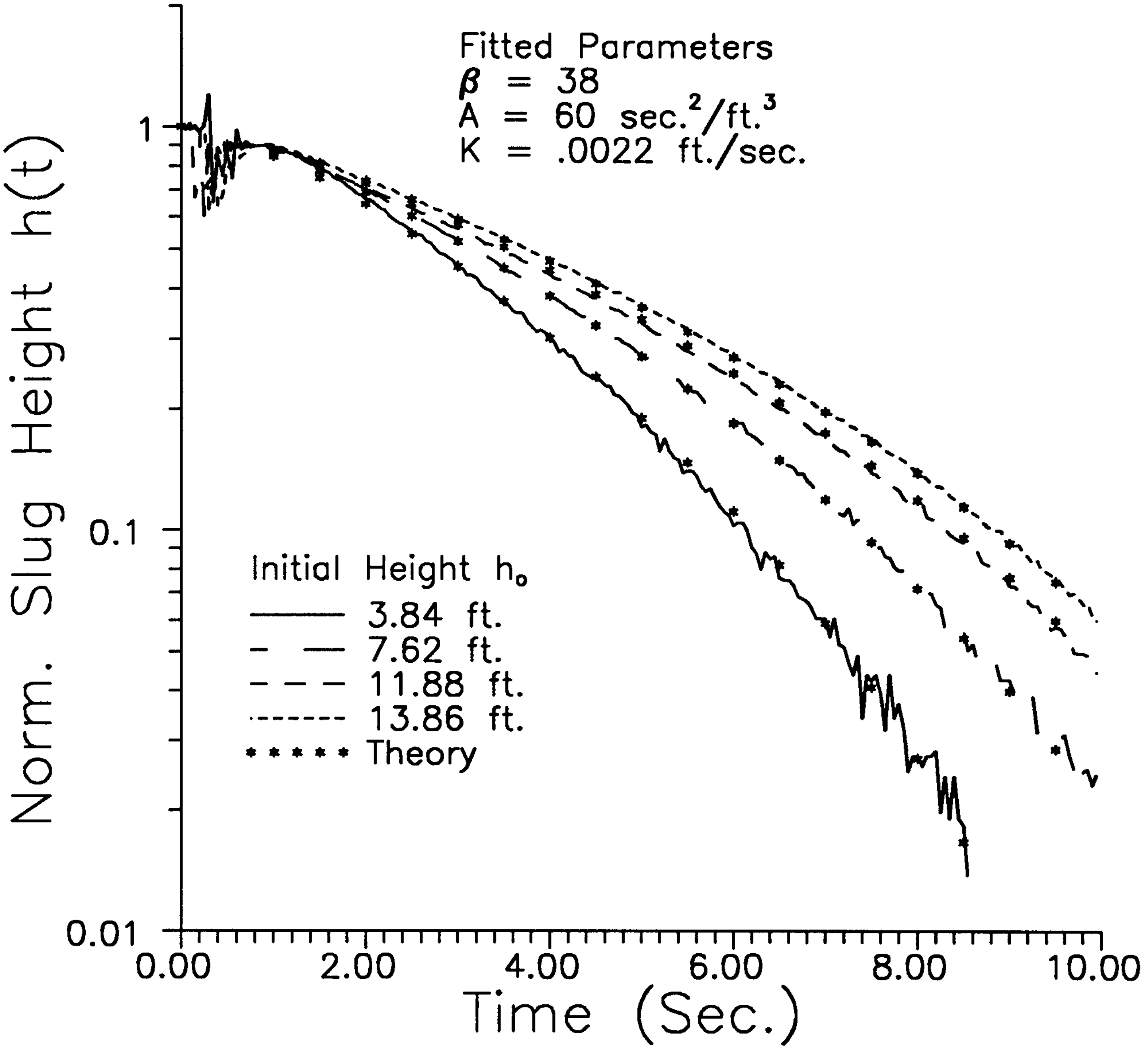


Figure 6.

Slug Test Response at GEMS Well 0-7

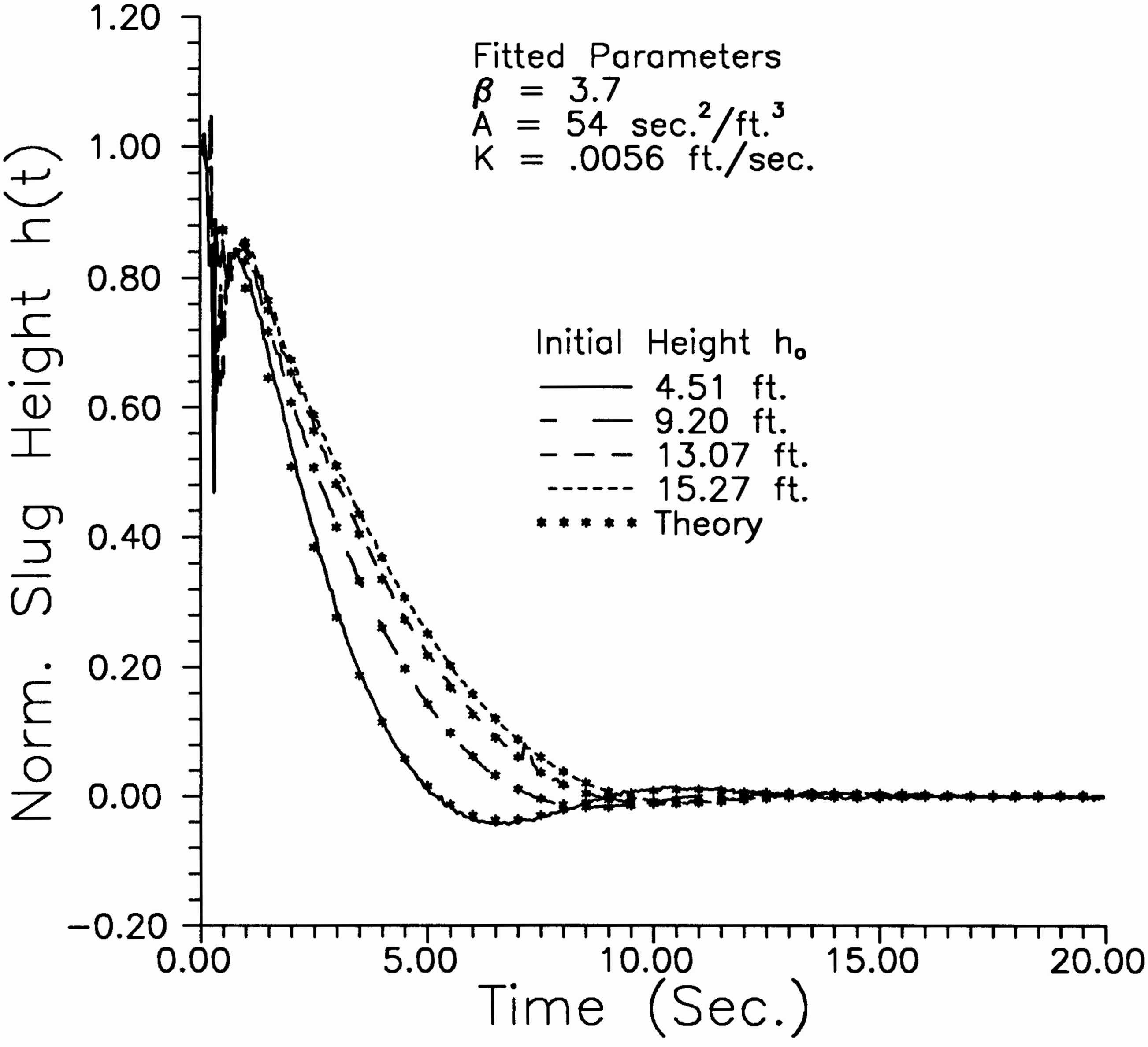


Figure 7.

Slug Test Response at GEMS Well 0-5

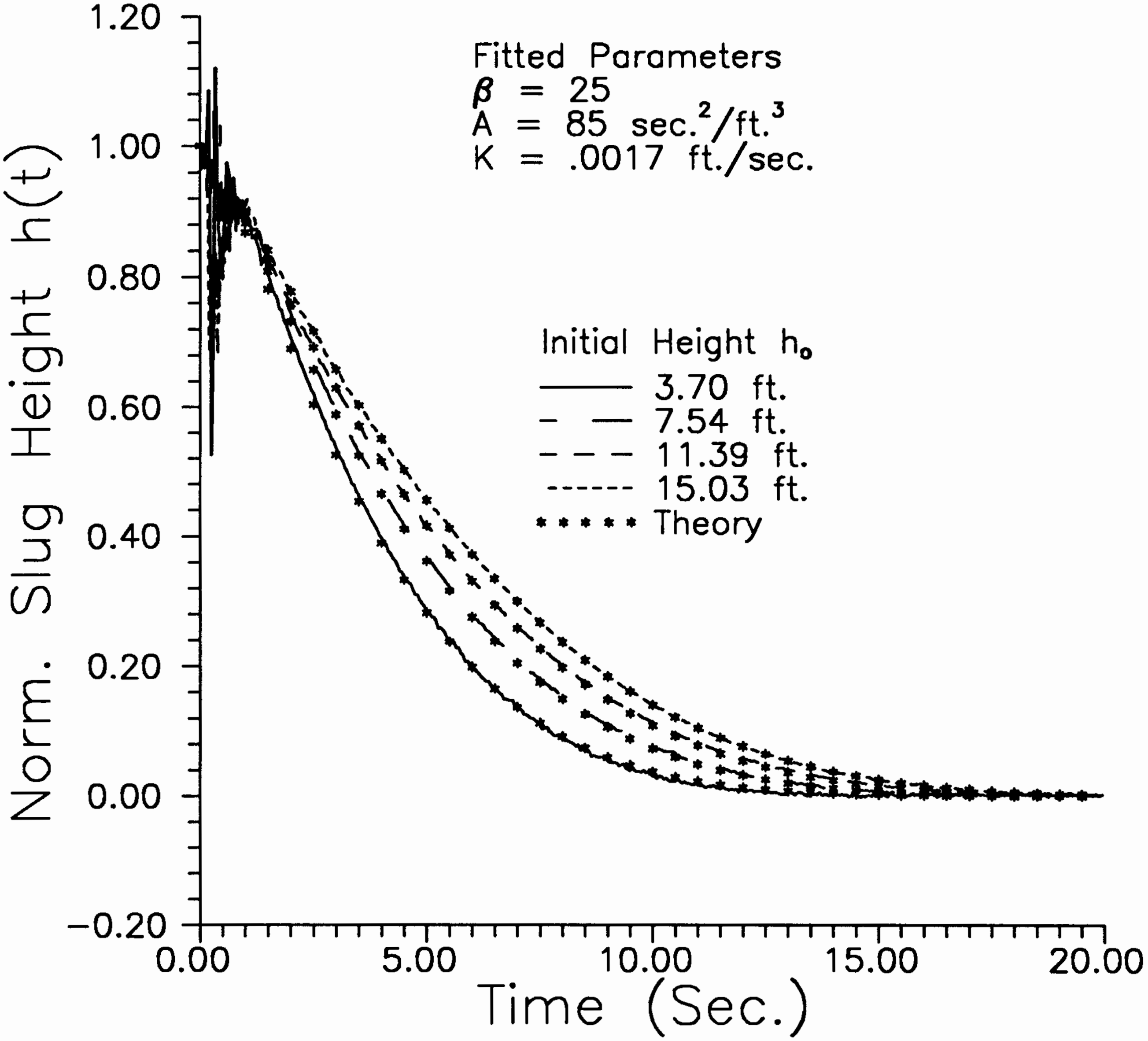


Figure 8.

Slug Test Response at GEMS Well 0-7
Time Comparison 1993 and 1996

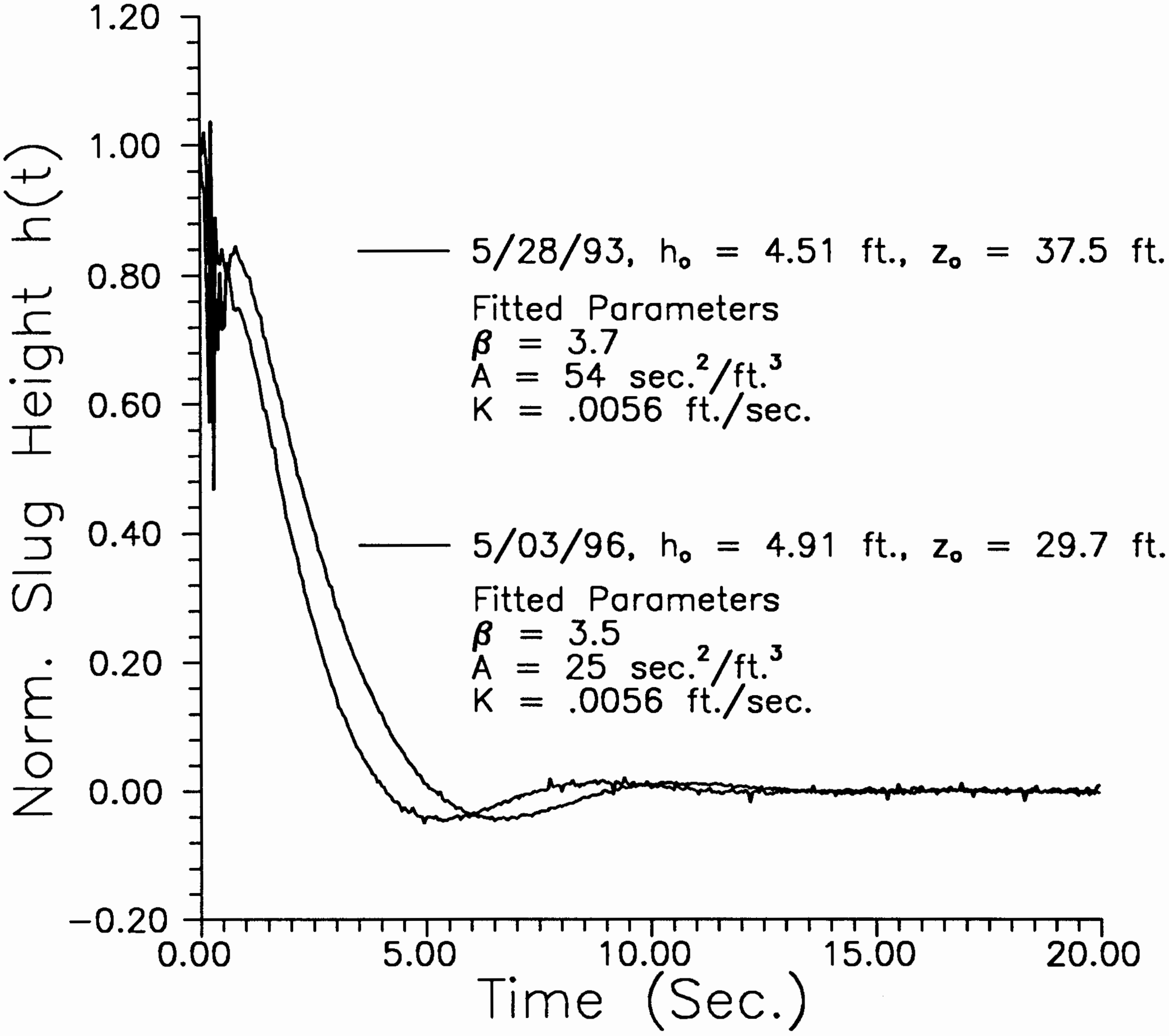


Figure 9.
Slug Test Response at GEMS Well 0–9
Casing Radius 4 Inches

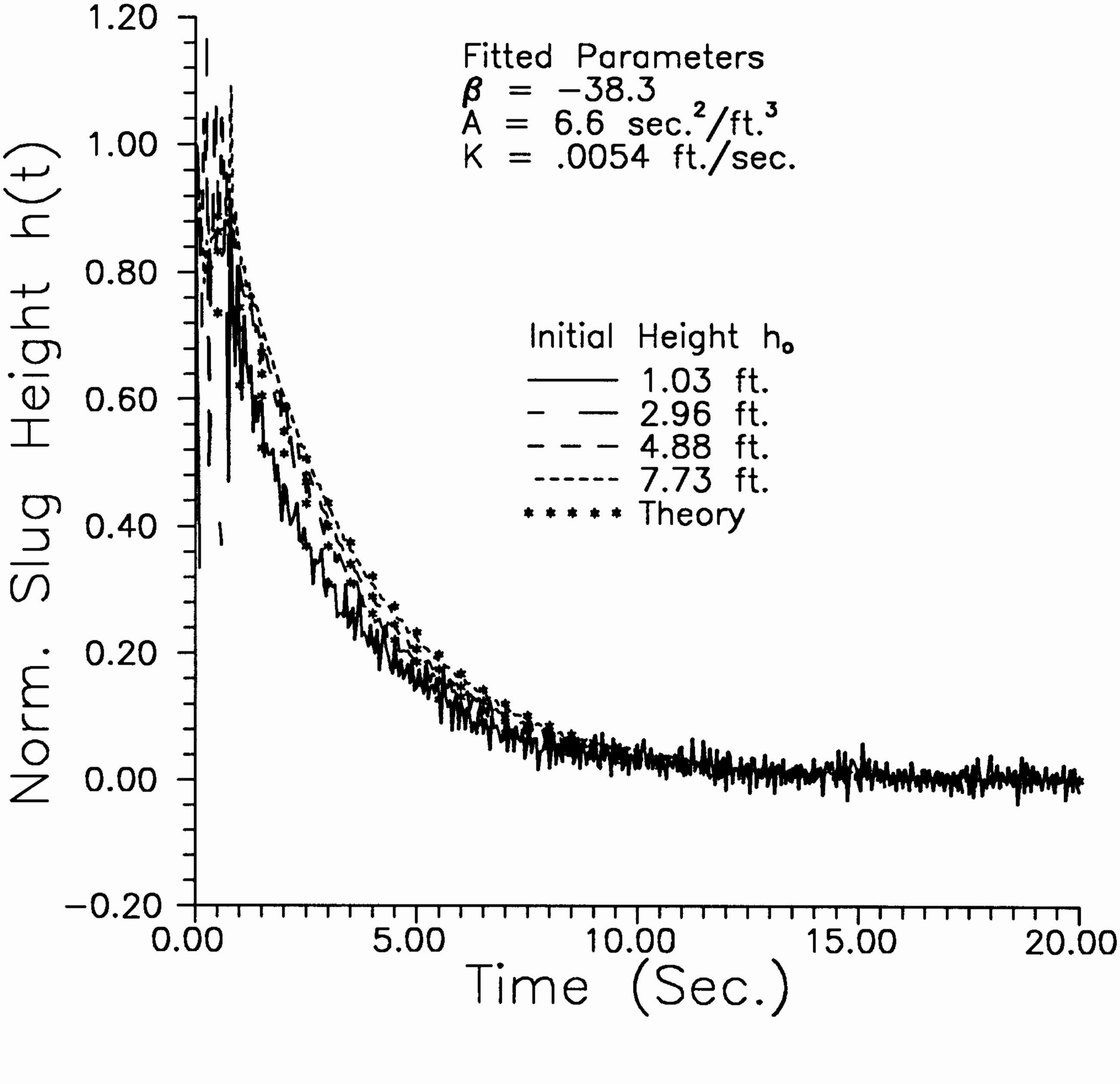


Figure 10.
Slug Test Response at GEMS Well 0-9
Casing Radius 2 Inches

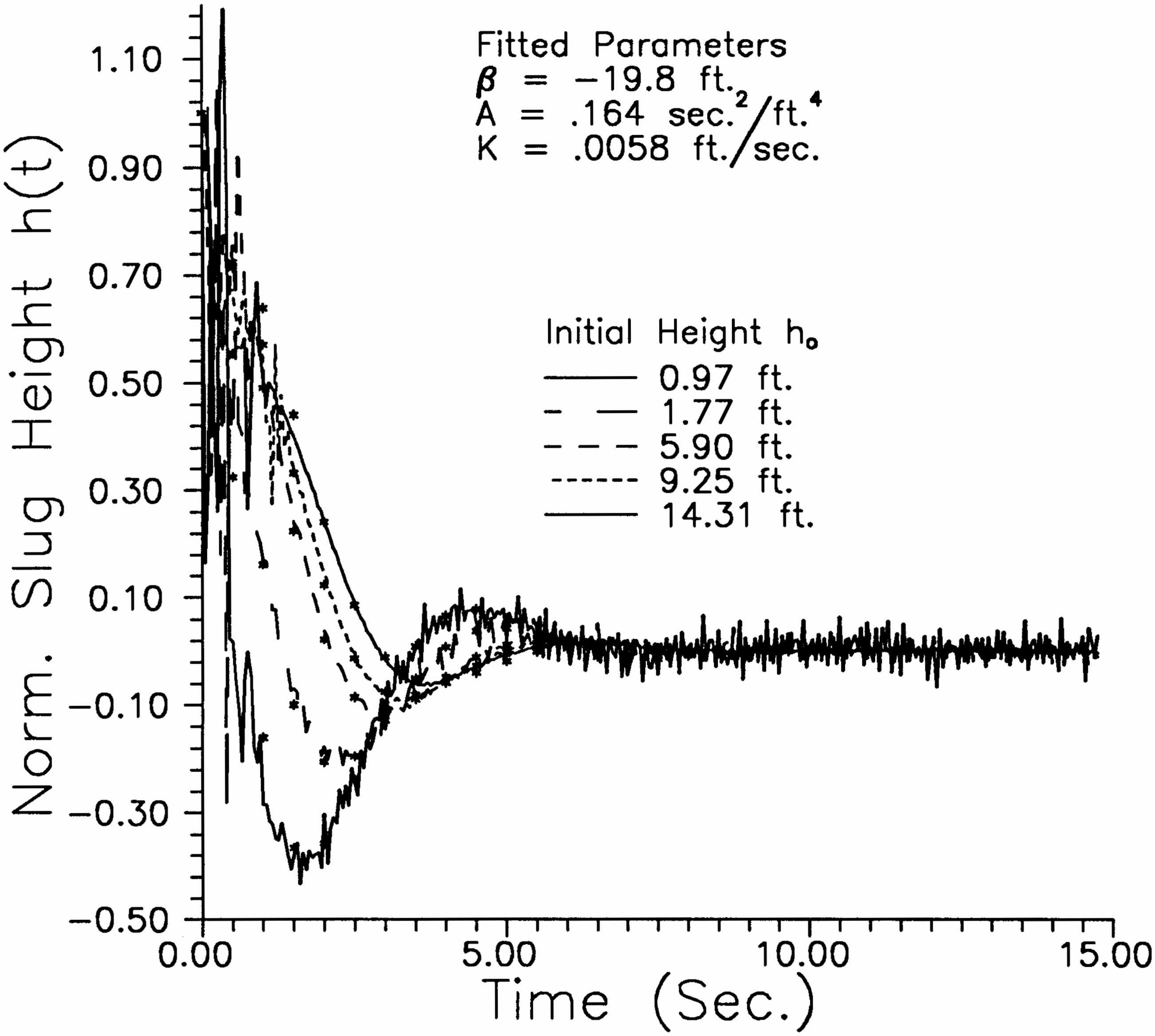


Figure 11.

Slug Test Response at GEMS Well 00-1

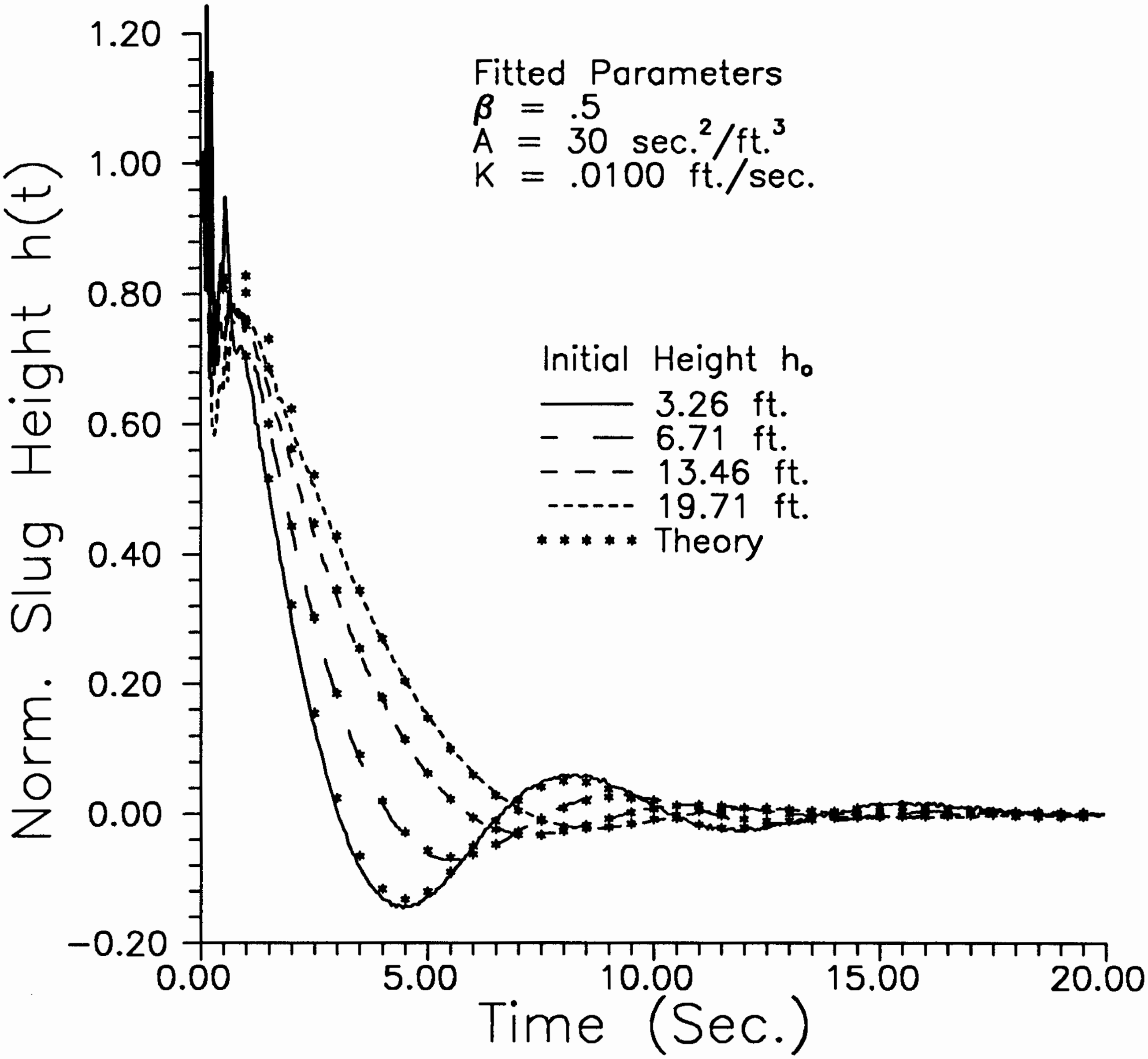


Figure 12.

Photograph of Two Inch Packer and Attachments

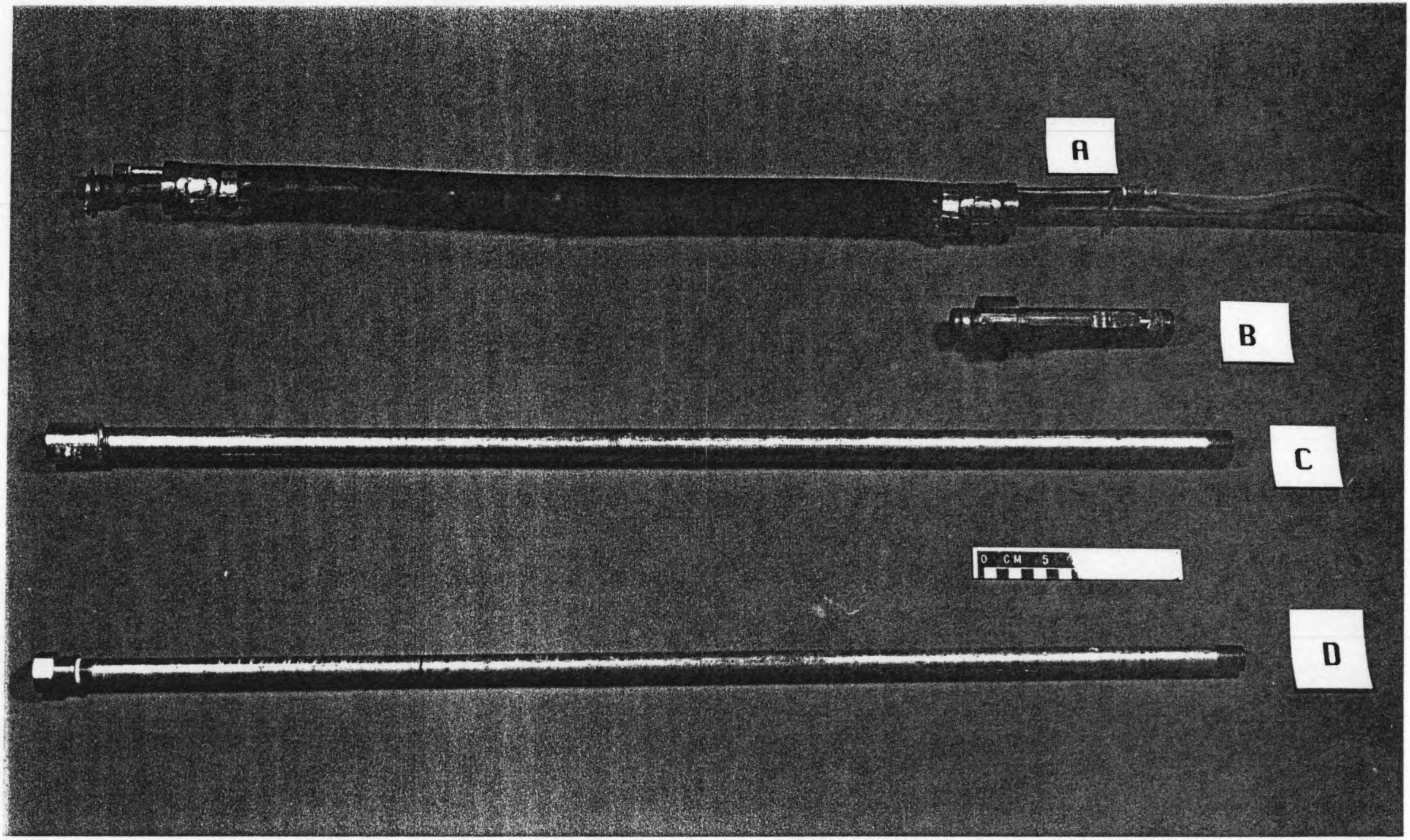

A
B
C
0 CM 5
D

Figure 13.

Slug Test Response at GEMS Well 00-1

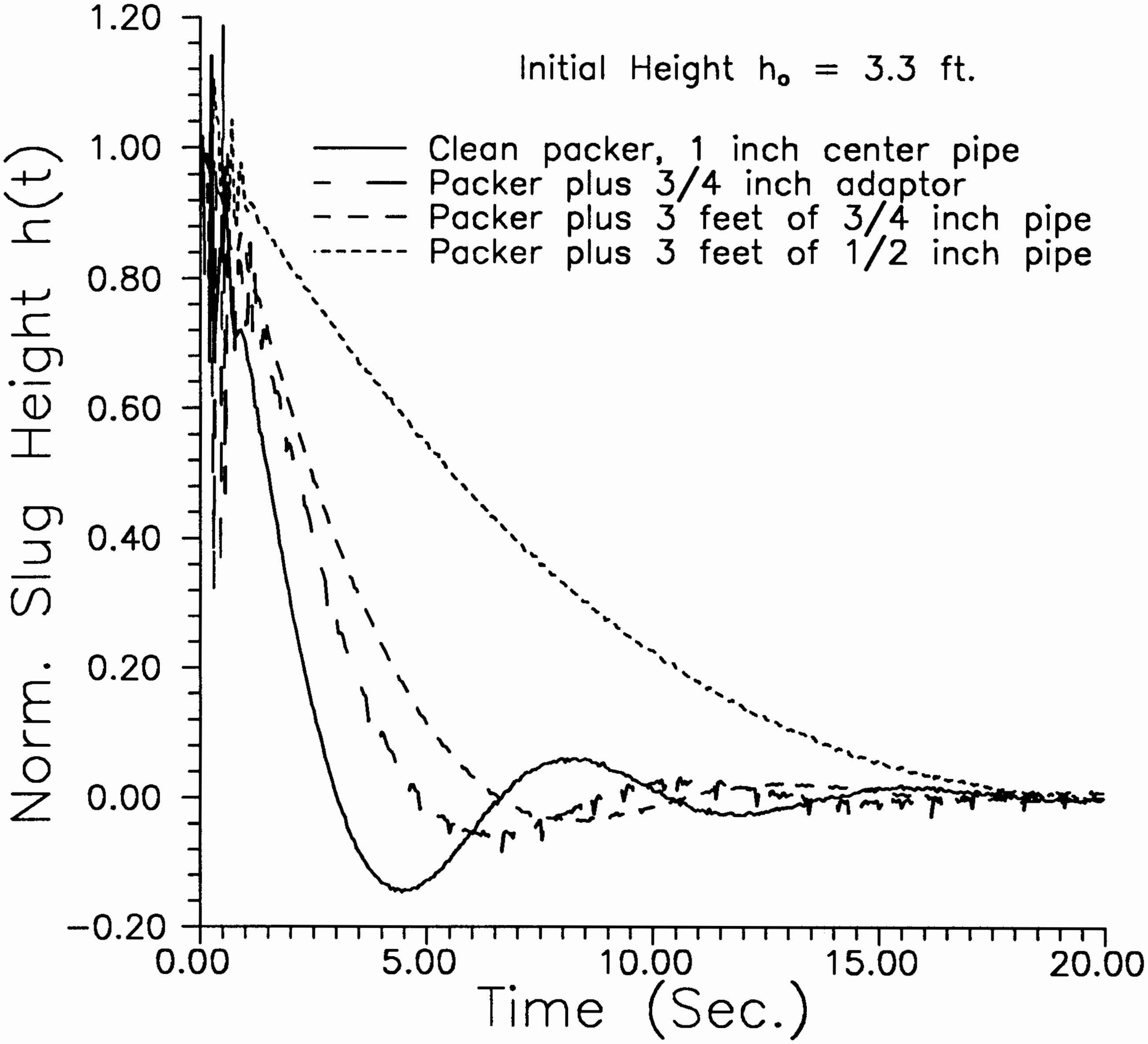
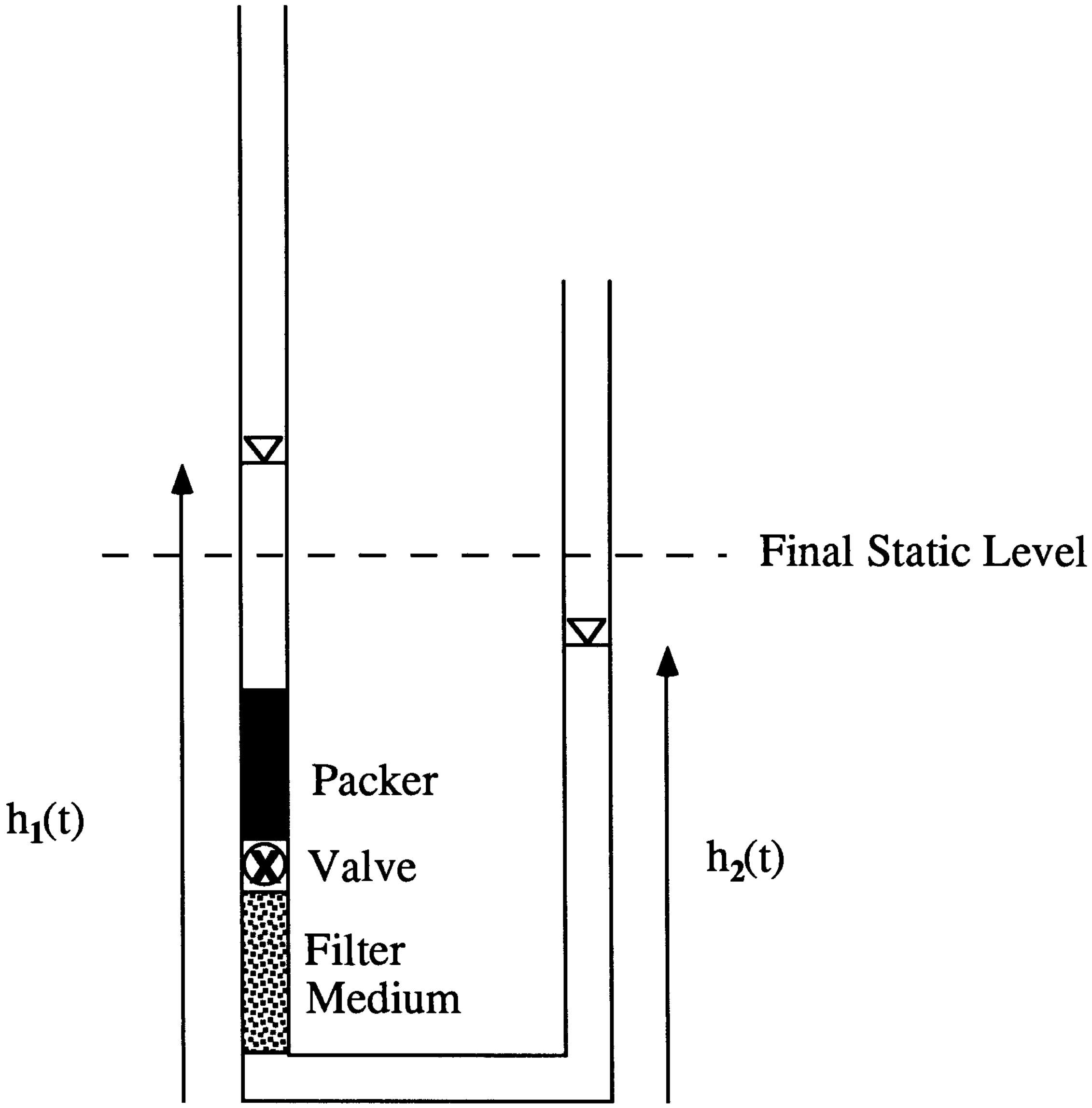


Figure 14.

Double Column Test Apparatus



$$h(t) = h_1(t) - h_2(t)$$

Figure 15.
Slug Response Double Column Test
No Packer and No Filter Medium